# Analysis of the DRVID and Dual Frequency Tracking Methods in the Presence of a Time-Varying Interplanetary Plasma

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An analysis is made of two different methods for determining the total electron content of the plasma existing between a spacecraft and the Earth. It is shown that the two methods complement each other. The dual frequency method is capable of measuring the structure of a plasma inhomogeneity, whereas the DRVID method is capable of locating this inhomogeneity within the ray path of the electromagnetic tracking signal.

## I. Derivation of the Pertinent Expressions

Suppose, an electromagnetic signal (either a continuous or a modulated wave) is propagating along the $x$ axis of a cartesian coordinate system toward a spacecraft. It will then interact with the interplanetary plasma. For the high frequency involved ($\omega \approx 10^{10}$ sec$^{-1}$) only the electrons of the plasma will contribute to the degradation of the signal. Assuming that the electron density $N$ depends only on $x$ and the time $t$, that is $N = N(x, t)$, it can be shown that the field equation for the amplitude of the electric field $E$ of the wave is given by

$$-V^2 E + \frac{1}{c^2}\ddot{E} = -\frac{\alpha}{c^2} N(x, t) E \qquad (1)$$

the electric field is polarized in a direction perpendicular to $x$, the direction of propagation. $\alpha = 4\pi e^2/m$. A term of the order of $\omega^{-1}\dot{N}$ has been neglected in Eq. (1), since surely

$$\omega^{-1}\dot{N} << N \qquad (2)$$

because time variations of $N$ are measured in minutes for typical solar plasma inhomogeneities (plasma clouds).

Equation (1) may be solved by the ansatz:

$$E = e^{i(kx+F(x, t)-\omega t)} \qquad (3)$$

Inserting Eq. (3) into Eq. (1), we obtain for $F$ the following equation:

$$\frac{\partial F}{\partial x} + \frac{1}{c}\frac{\partial F}{\partial t} = -\frac{\alpha}{2kc^2} N(x, t) \qquad (4)$$

neglecting $(dF/dx)^2$ and $d^2F/dx^2$, since it can be shown that these terms are small compared to $k^2$. The solution of Eq. (4) for $F = 0$ when $N \equiv 0$, is

$$F = -\frac{\alpha}{2kc^2}\int_C^x N\left(x', t + \frac{x' - x}{c}\right) dx' \qquad (5)$$

with an as yet undetermined integration constant $C$.

A phase point (wave crest) moves with a velocity $dx/dt$ obtained from

$$kx - \frac{\alpha}{2kc^2} \int_C^x dx' N\left(x', t + \frac{x'-x}{c}\right) - \omega t = 0 \qquad (6)$$

If at $t = 0$ the distance $x$ traveled by the phase point is zero, or in other words the wave crest in question is emitted at $t = 0$, then $C = 0$ and the integration constant is determined. Neglecting $N$ in Eq. (6) leads to

$$x = ct, \quad \text{or} \quad \frac{dx}{dt} = c, \qquad (7)$$

the vacuum value for the phase velocity. But since the derivative of the integral in Eq. (6) is small compared to $k$ we may use Eq. (7) for the time argument of $N$ in Eq. (6). Anything better would only lead to second-order effects which have been neglected here in the first place. Accordingly, the phase may be expressed by:

$$kx - \frac{\alpha}{2kc^2} \int_0^x N\left(x', \frac{x'}{c}\right) dx' - \omega t = 0 \qquad (8)$$

from which it follows that the instantaneous phase velocity at $x$ and $t = t' = x/c$ given by

$$\dot{x} = v_P(x, t) = c\left(1 + \frac{\alpha}{2\omega^2} N(x, t')\right) \qquad (9)$$

as intuitively expected. For rapid fluctuations of $N$ and low frequencies, however, the result (9) would be totally inadequate. The group velocity is also given by Eq. (9) if $\alpha$ is interchanged with $-\alpha$.

Let us now suppose that a particular wave crest is emitted from the antenna at position $x = 0$ at time $t$. Let us also suppose that the *true* distance to the spacecraft, the range, is $R$ (in meters). Then the transit time $T_U$ (U for uplink) for arrival at the spacecraft is

$$T_U = \int_0^R \frac{dx}{v_P\left(x, \frac{x}{c} + t\right)}$$

$$= \frac{R}{c}\left(1 - \frac{\alpha}{2\omega^2} \frac{1}{R} \int_0^R N\left(x, \frac{x}{c} + t\right) dx\right) \qquad (10)$$

Likewise, for the downlink transit time $T_D$ for arrival at the earthbound receiver we have

$$T_D = \frac{R}{c}\left(1 - \frac{\alpha}{2\omega_D^2} \frac{1}{R} \int_0^R N\left(x, T_D + T_U - \frac{x}{c} + t\right) dx\right) \qquad (11)$$

where $\omega_D$ is the transponder frequency of the spacecraft (usually $240/(221)\omega$). The total roundtrip time is $T_U + T_D$ and the apparent range is defined as:

$$R' = \frac{c}{2}(T_U - T_D) \qquad (12)$$

Eq. (10) and (11) apply to the phase velocity. For the group velocity of the modulated signal Eqs. (10) and (11) apply if $\alpha$ is replaced by $-\alpha$, as already stated. From Eqs. (10), (11), and (12) we find

$$R' = R - \frac{\alpha}{4\omega^2} \int_0^R \left[ N\left(x, \frac{x}{c} + t\right) \right.$$
$$\left. + \left(\frac{\omega}{\omega_D}\right)^2 N\left(x, T_D + T_U - \frac{x}{c} + t\right) \right] dx \qquad (13)$$

The apparent range for the modulation (group velocity) is

$$R'' = R + \frac{\alpha}{4\omega^2} \text{(integral of Eq. 13)} \qquad (14)$$

This is true, since the time $T_U$ and $T_D$ in the argument of the electron density may be expressed by $c^{-1}R$, $c^{-1}R'$ or $c^{-1}R''$, the difference between any of these choices being of second order. Forming the "differenced range minus integrated doppler" (Ref. 1) we have the equation[1]:

$$\text{DRVID}(t) = R''(t) - R'(t) = \frac{\alpha}{2\omega^2} \int_0^R \left[ N\left(x, \frac{x}{c} + t\right) \right.$$
$$\left. + \left(\frac{\omega}{\omega_D}\right)^2 N\left(x, \frac{2R-x}{c} + t\right) \right] dx \qquad (15)$$

The quantity needed to correct the doppler data (phase velocity) is then

$$-\frac{1}{2}\frac{d}{dt}\text{DRVID}(t) = -\frac{1}{2}\frac{d}{dt}(R''(t) - R'(t)) \qquad (16)$$

[1]Disregarding the doppler shift which is trivially incorporated, if needed.

We turn now to the dual-frequency method extensively used by the Stanford group (e.g., Ref. 2). The method consists in transmitting simultaneously two signals at two different frequencies along the ray path toward the spacecraft. The delay time difference is measured at the spacecraft and telemetered back to earth. From Eq. (1) we find for the time difference:

$$\Delta T = T_{v_1} - T_{v_2} = -\frac{\alpha}{2c}\left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right)\int_0^R N\left(x, \frac{x}{c} + t\right) dx$$

(17)

Where $\omega_1$ and $\omega_2$ are the two frequencies (usually 50 and 425 MHz). The apparent range change is then for the dual-frequency method (DFM)

$$DFM(t) = c\Delta T$$

(18)

If the group velocity is employed, the sign of $\alpha$ changes in Eqs. (17) and (18). We see that the DFM measures only the uplink electron content. But from Eqs. (15) and (18) we obtain

$$DRVID(t) + \frac{\omega_1^2\omega_2^2}{\omega^2(\omega_1^2 + \omega_2^2)} DFM(t) =$$

$$\frac{\alpha}{2}\left(\frac{\omega}{\omega_0}\right)^2\int_0^R N\left(x, \frac{2R - x}{c} + t\right) dx$$

(19)

which is a measure of the downlink electron content alone. There is, however, no new information in Eq. (19), since it is the one way information of Eq. (18) shifted by $R/c$ in time. From Eq. (15) it is obvious that if the electron content changes significantly in times short compared to the transit time of the signal, the analysis of the interplanetary plasma is hampered by the occurrence of the two integrals in Eq. (15), one being shifted in time by the amount $R/c$, the transit time. Large and fairly quick electron content changes can happen when a plasma cloud emitted from an active area of the sun passes the raypath (Refs. 3 and 4).

## II. Application

To see more clearly what is involved, let us take a plasma cloud of length $L$ which is entering the ray path at $t = 0$. We also assume for simplicity that the width of the cloud is short compared to the range $R$ and that therefore the transit time through the cloud is negligibly small compared to $R/c$. Further, we assume a parabolic electron density distribution within the cloud with maximum

at the center. If this cloud enters the raypath at $x = x_0$, it may conveniently be described by the following expression:

$$N(x,t) = LN_0\,\delta(x - x_0)\left[1 - \frac{4}{L^2}\left(\frac{L}{2} - vt\right)^2\right]$$

$$\times\left[S(t) - S\left(t - \frac{L}{v}\right)\right]$$

(20)

Here $\delta(x)$ is the delta function and $S(t)$ the step function

$$S(t) = \begin{cases} 0 \text{ for } t < 0 \\ 1 \text{ for } t > 0 \end{cases}$$

(21)

To simplify the analysis, let us investigate the integrals

$$I_2 = \int_0^R\left[N\left(x, \frac{x}{c} + t\right) + N\left(x, \frac{2R - x}{c} + t\right)\right] dx$$

(22)

representative for the DRVID data, Eq. (15) and

$$I_1 = \int_0^R N\left(x, \frac{x}{c} + t\right) dx$$

(23)

representative for the DFM data, Eq. (17).

With the plasma cloud given by Eq. (2) we have

$$I_1 = LN_0\left[1 - \frac{4}{L^2}\left(\frac{L}{2} - v\left(\frac{x_0}{c} + t\right)\right)^2\right]$$

$$\times\left[S\left(\frac{x_0}{c} + t\right) - S\left(\frac{x_0}{c} + t - \frac{L}{v}\right)\right]$$

(24)

and

$$I_2 = I_1 + LN_0\left[1 - \frac{4}{L^2}\left(\frac{L}{2} - v\left(\frac{2R - x_0}{c} + t\right)\right)^2\right]$$

$$\times\left[S\left(\frac{2R - x_0}{c} + t\right) - S\left(\frac{2R - x_0}{c} + t - \frac{L}{v}\right)\right]$$

(25)

Four examples of Eq. (24) and (25) are depicted in Fig. 1. By and large, characteristic values of the various parameters have been chosen for the computation of these curves. It is clear from these figures that the time rate of change of the electron density within the plasma cloud is measured differently between DRVID and DFM. Of course, we know from our previous analysis that the DFM data give a direct measure of electron concentration time variation, whereas DRVID degrades this information.

Let us look more carefully at the statement just made. We strip the analysis from all unessentials such as constant factors, etc. The conclusion is, simply, that DRVID measures the quantity:

$$\Phi(t) = N(t) + N(t + \delta) \tag{26}$$

where $N(t)$ is proportional to the electron density of the plasma inhomogeneity. The DFM however, measures

$$\Psi(t) = N(t) \tag{27}$$

and, therefore, gives a direct measurement of the structure of the plasma cloud. Now, it is important to notice that the delay time $\delta$ of Eq. (26) actually can be measured by DRVID. This delay time determines the location of the cloud within the ray path. The DFM is incapable of furnishing this information. The determination of the location of a plasma cloud in the ray path is done by an autocorrelation analysis of the tracking data, or what it amounts to in our simplified analysis, the quantity $\Phi(t)$ of Eq. (26). For details see Ref. 3. However, even with the knowledge of $\delta$ in Eq. (26), the determination of $N(t)$ meets with difficulties. For, if $\Phi(t)$ is known and $\delta$ is known, the solution of Eq. (26) is

$$N(t) = a \sum_{n=1}^{\infty} (-1)^{n+1} \Phi(t - n\delta)$$

$$+ (1 - a) \sum_{n=0}^{\infty} (-1)^n \Phi(t + n\delta) \tag{28}$$

where $a$ is an integration constant to be determined by the initial conditions (no cloud prior to a certain time for instance). In any case, we see that the determination of $N(t)$ hinges on values of $\Phi(t)$ measured at many different times. If $\epsilon$ is the error of a measurement of $\Phi(t)$ and if $n$ values of $\Phi(t)$ are needed in each of the sums of Eq. (28), the error in the determination of $N(t)$ would roughly be $\epsilon\sqrt{2n}$. Assuming that the DFM has the same accuracy as DRVID, the error in determining the solar plasma cloud structure would only be $\epsilon$, which may be considerably less than the accuracy of the DRVID cloud structure determination. For $\delta = 4$ min and the passage of the cloud through the ray path lasting 40 min, we need about $2 \cdot 40/4 = 20$ terms in Eq. (28). The inaccuracy of Eq. (26) versus Eq. (27) would be 4.5 times worse in this case. We

have not mentioned the inaccuracy in the value of $\delta$. This brings in additional errors in the cloud structure determination by DRVID which are difficult to ascertain.

There is, however, another method for determining $\delta$, assuming only one cloud is present, that avoids the correlation analysis mentioned earlier. It consists simply in the following: There is a point in time when DRVID ceases to pick up information from the cloud on the downlink. This happens when the round-trip time from the cloud to the spacecraft and back is equal to the time for the cloud to just have left the ray path. This time is given by

$$2\frac{R - x_0}{c} = \frac{L_1}{v} \tag{29}$$

where $L_1$ is that part of the cloud that affects only the uplink information. DRVID and DFM become identical (Eqs. 22 and 23). But the time $L_1/v$ is equal to the time difference between the time DRVID and DFM become equal and the time the cloud ceases to affect the data. $L_1/v$ can therefore be measured. Knowing the range it is now easy to compute $x_0$, i.e., the location of the tail end of the cloud. A similar analysis shows that the location of the front end of the cloud may be determined by Eq. (29) where now $L_1/v$ is the time difference between the time the cloud starts to affect DRVID and the time it starts to be seen by DFM.

## III. Summary

To summarize this analysis, we wish to make the following statements: Given that the inherent accuracy of the DFM and DRVID are the same, and there is no reason to believe otherwise, the two methods complement each other nicely. Whereas DRVID alone is capable of locating solar plasma inhomogeneities, the DFM alone is capable of determining the structure (that is, the spatial electron density variations) of a solar plasma inhomogeneity. It is felt therefore, that if both methods are available, a great deal more can be learned about the solar wind of which so pitifully little is known to this date.

# References

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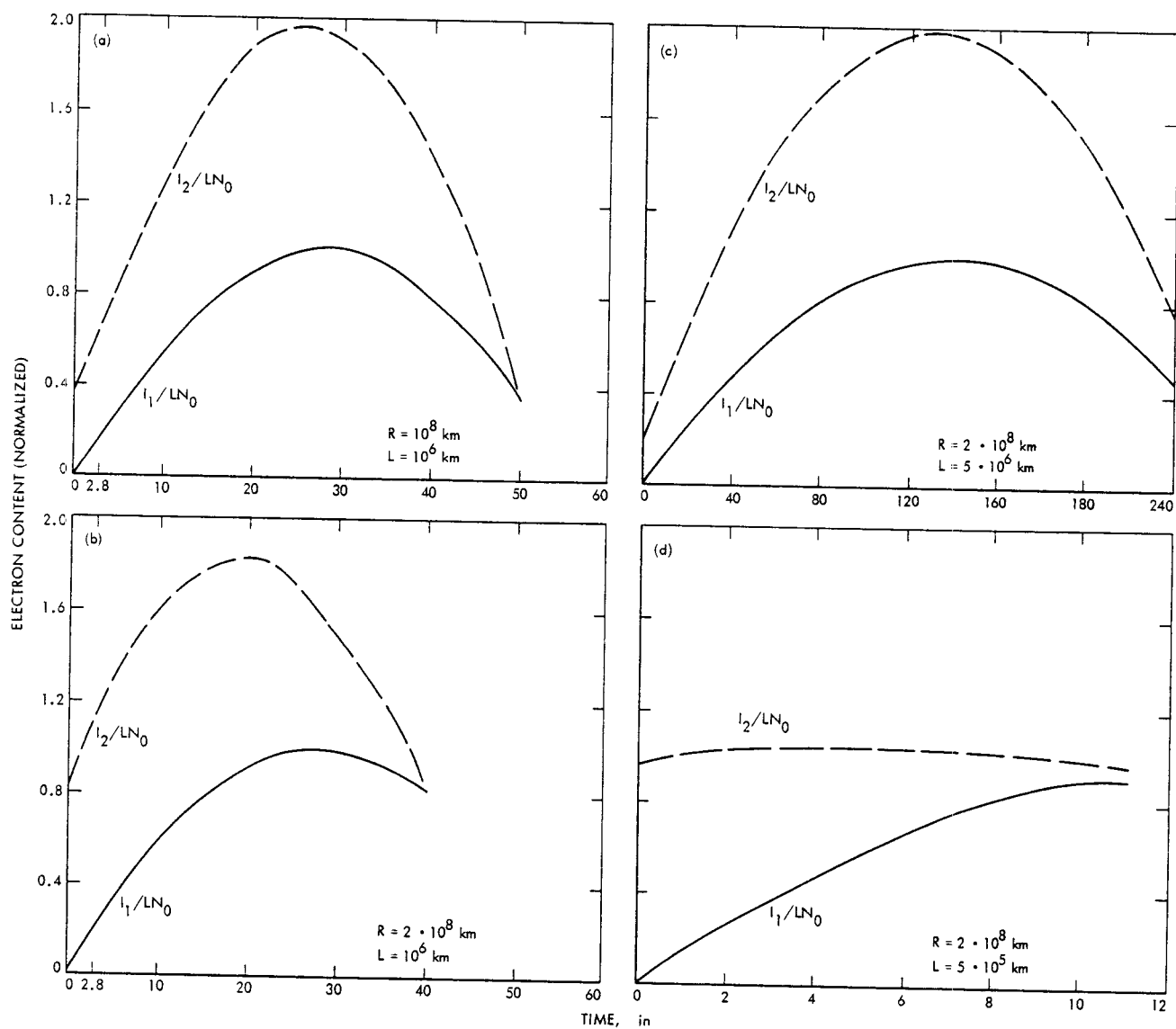4. Koehler, R. L., *J. Geophys. Res.*, Vol. 73, p. 4883, 1968.

Fig. 1. Four selected solutions of Eqs. (24) and (25): (a) $R = 10^8$ km and $L = 10^6$ km, (b) $R = 2.10^8$ km and $L = 10^6$ km, (c) $R = 2 \times 10^8$ km and $L = 5 \times 10^6$ km, and (d) $R = 2.10^8$ km and $L = 5 \times 10^5$ km